Thermo-structural analysis of reinforced concrete beams

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Abstract

The objective of this study is to simulate the behavior of reinforced concrete beams in fire situation. In order to achieve this objective, advanced numerical formulations were developed, implemented and evaluated. When exposed to high temperatures, the properties of the material deteriorate, resulting in the loss of strength and stiffness. To achieve the goal, two new modules within the Computational System for Advanced Structural Analysis were created: Fire Analysis and Fire Structural Analysis. The first one aims to determine the temperature field in the cross section of structural elements through thermal analysis by using the Finite Element Method (FEM). The second was designed to perform the second-order inelastic analysis of structures under fire using FEM formulations based on the Refined Plastic Hinge Method coupled with the Strain Compatibility Method. The results obtained of the nonlinear analyses of two reinforced concrete beams under high temperature were compared with the numerical and experimental solutions available in literature and were highly satisfactory. These results also showed that the proposed numerical approach can be used to study the progressive collapse of other reinforced concrete structures in fire situation and extended to the numerical analysis of composite structures under fire condition.

Introduction

A thermo-structural analysis basically consists of determining the temperature distribution in the structural elements (thermal analysis) and simulating the displacements, deformations and internal stresses in the structure (structural analysis), considering the loss of strength and stiffness of the structural material due to temperature increase. Increasingly sophisticated analytical, numerical and experimental models, which allow a better knowledge and understanding of the structural behavior and construction materials in a fire situation, have been constantly proposed in the scientific environment. However, the high cost of experimental tests has provoked a trend in research toward numerical analysis of structures in a fire situation.

Advanced models for heat transfer problems often refer to computational models. In most fire structural engineering problems, thermal analysis is transient, with time-dependent boundary conditions, and temperature-dependent material properties, which gives such analysis a considerably nonlinear character. In the context of thermo-structural analysis, with the temperature field established in thermal analysis, the variation of mechanical and thermal properties of the material as a function of temperature are considered in structural analysis, thus establishing a weak coupling between these analyses (thermal and structural). Some research involving thermo-structural analysis of beams and other reinforced concrete structural systems can be highlighted in literature. Among other structural systems, seven beams were designed in the study of Ba *et al.*[1] to create different standard cracks and investigate heat propagation in reinforced concrete structures. The commercially available finite element computational program ANSYS was adopted to describe the thermal behavior of heated reinforced concrete structures and to evaluate the cracking effect. Both experimental and numerical results showed that heat spread tended to increase in cracked regions compared to undamaged regions.

In the study by Ruzic *et al.*,[2] a new partially coupled numerical finite element model is shown based on evaluating the fire resistance of curved reinforced concrete beams subjected to concrete fragmentation, spalling. The idea of the numerical model is to evaluate the time and magnitude of concrete fragmentation in fire conditions in these beams, which are commonly used in tunnels. The authors point out that the study indicates that the adoption of partially coupled or fully coupled models is obligatory in order to properly assess the structural safety of fire exposed reinforced concrete structures.

Ahmad[3] investigated the effect of different total and partial fire exposure conditions on the behavior of reinforced concrete beams. Six reinforced concrete beams were tested under bending after one hour of exposure to standard fire, followed by a standard cooling phase. The results showed that the exposure of the different faces to the fire significantly affects the load of the first crack and the deflection behavior. A numerical procedure to predict the mechanical behavior of fire-exposed reinforced concrete beams is also shown in Sun *et al.*[4] In this study, the authors capture the thermal behavior through a two-dimensional finite element model, while a one-dimensional numerical model is formulated to simulate the mechanical response of beams under increasing loads. The authors point out that few elements are used in the analysis and that the numerical results are consistent with the experimental test data, which demonstrates the model's ability to effi-

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Key words: Thermal analysis; Thermo-structural analysis; Fire; Finite Element Method; Computational System for Advanced Structural Analysis.

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ciently simulate the thermomechanical behavior of reinforced concrete beams.

More recently, six full-scale simply supported reinforced concrete beams were designed according to the *strong bending and weak shearing* principle by Song *et al.*[5] to investigate three-sided exposed fire resistance behavior. One beam was analyzed at room temperature while the other five were exposed to high temperature. Experimental results showed that weak shearing failures at room temperature can be transferred to high temperature bending failures due to thermal expansion and the degradation of concrete and steel strength. The authors pointed out that the higher the longitudinal reinforcement ratio, the longer the beam failure time, indicating that the longitudinal reinforcement fixation action can significantly improve the beams shear capacity under high temperature. In addition, the stirrup reinforcement configuration can effectively reduce the fragile change in vertical deflection when the beam enters the failure stage. More studies involving reinforced concrete beams in a fire situation can be highlighted, such as the articles by Albuquerque and Silva,[6] Raouffard and Nishiyama[7] and Simões and Santos.[8]

In this context, in the present study, the objective is to perform the thermo-mechanical analysis of reinforced concrete beams, subjected to high temperatures, using the Strain Compatibility Method (SCM)[9-11] associated with the Refined Plastic Hinge Method (RPHM). For this, two computational modules were developed and coupled to the Computational System for Advanced Structural Analysis (CS-ASA) program. The first one, called CS-ASA/Fire Analysis (CS-ASA/FA)[12,13] is able to determine the temperature field in the cross-section of structural elements through thermal analysis via Finite Element Method (FEM) in permanent and transient regimes. The second module, CS-ASA/Fire Structural Analysis (CS-ASA/FSA),[14-16] was developed to perform a second order inelastic analysis of structures under high temperatures. Thus, a SCM-based approach is proposed for the evaluation of the cross-sectional strength, axial stiffness and flexural strength of reinforced concrete beams under high temperatures. Thus, the coupling of this methodology with RPHM is aimed at evaluating plasticity in nodal terms through generalized stiffness parameters.

It is important to emphasize that the implemented computer system products include: a complete cross-sectional analysis, *i.e.* knowledge of the cross-sectional temperature field for any or all faces under fire exposure (high temperatures), as well as the load capacity (interaction diagrams) of the cross section for any or all faces under fire exposure; and a complete thermo-mechanical analysis, *i.e.* obtaining the structural members and displacements of the system and varying the internal forces with increasing temperature. The results of nonlinear fire analysis obtained herein for reinforced concrete beams were compared with the numerical and experimental solutions available in literature and were extremely satisfactory.

Materials and Methods

Thermal-structural analysis

Herein, a procedure for the consecutive resolution of two systems of equations within each time interval is adopted for analysis of structures in fire situation. A system results from the integration of the heat conduction equation (thermal analysis); the other system corresponds to the incremental equilibrium equations (structural analysis). Thus, attention is given to the analysis of reinforced concrete beams through the CS-ASA program which has been expanded with two new modules: CS-ASA/FA and CS-ASA/FSA.

Computational System for Advanced Structural Analysis/Fire Analysis

It is assumed in this study that the temperature distribution along each structural element is uniform and equal to that estimated for the cross section. Thermal analysis is then performed exclusively in the cross-sectional plane through numerical heat transfer models that enable the determination of the temperature distribution at different points of the section. A time integration strategy based on the Finite Difference Method is adopted. The CS-ASA/FA module also has two system resolution procedures: simple incremental and iterative incremental (Picard or Newton-Raphson). It is also worth mentioning that the thermal and mechanical properties of steel and concrete in fire conditions are adopted according to normative specifications.[17,18] Further details of this computational module are in Maximiano[19] and Barros *et al.*[14]

Computational System for Advanced Structural Analysis/Fire Structural Analysis

For the study of the inelastic behavior of reinforced concrete structures in a fire situation, the RPHM coupled with SCM is used as previously mentioned. These numerical strategies were adapted in order to consider the effects of the degradation of the stiffness and strength parameters of the material, as well as the influence of thermal strain in the analysis of the element cross-section.

Finite element formulation by Refined Plastic Hinge Method

The CS-ASA/FSA module uses a RPHM-based formulation to simulate concentrated plasticity at the nodal points of the element. Therefore, it is assumed that: i) all elements are initially straight and prismatic, and the cross-section remains flat after deformation; ii) the effects of local instability are neglected; iii) the structure is perfectly locked on the orthogonal axis (2D problem); iv) large displacements and rigid body rotations are allowed; and v) shear deformations effects are ignored.

For the adopted formulation, in the context of the discretization of the structural system via FEM, the reference co-rotational system is considered, where the finite element is delimited by nodal points i and j (Figure 1). In this same Figure, there are the internal forces acting on the element, M_i, M_j and P, as well as the respective degrees of freedom θ_i, θ_j and δ.

The equilibrium ratio of the finite element shown in Figure 1, in incremental form, is given by:

$$\begin{Bmatrix} \Delta P \\ \Delta M_i \\ \Delta M_j \end{Bmatrix} = \begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & k_{23} \\ 0 & k_{32} & k_{33} \end{bmatrix} \begin{Bmatrix} \Delta\delta \\ \Delta\theta_i \\ \Delta\theta_j \end{Bmatrix} \quad \text{Eq. 1}$$

where ΔP, ΔM_i and ΔM_j are the axial force and bending moment increments and $\Delta\delta$, $\Delta\theta_i$ and $\Delta\theta_j$ are the axial strain increments and nodal rotations, respectively. The terms related to the bending stiffness depend on the non-

Figure 1. Beam-column element in the co-rotational system.

linear geometric formulation, and the second order formulation proposed by Yang and Kuo[20] has been adopted herein. Expressions are developed to evaluate these terms of the stiffness matrix, considering a linear variation of the elasticity modulus along the length.[9,19]

Strain compatibility method

By subjecting a structural element to external forces, it deforms, generating internal forces to balance the system.[9] This strain at the cross-section level is addressed in the SCM. For the application of this method, it is assumed that the deformation field is linear and the section remains plane after deformation (Figure 2).

This method seeks to match the deformed configuration of the section to the acting forces. A form of discretization capable of capturing deformations as efficiently as possible should then be defined. In this study, a division of the cross-section area into fibers was adopted.[9,19] Regarding the constitutive relationships of materials, herein we used the relationships provided in Eurocode.[18] Figure 3 shows the stress-strain relationship adopted for concrete with increasing temperature. The behavior of the stress-strain diagram as a function of the temperature of the steel bars can be seen in detail in Maximiano.[19]

The Newton-Raphson iterative method is used in a cross-sectional application to obtain the moment-curvature ratio (M-ϕ). The discretization of the fiber cross section aims to describe the distribution of deformations by capturing the axial deformation (ε_i) in the plastic centroid (PC) of each fiber and then, through the constitutive relationships of the materials, obtain the respective stresses (σ_i). Thus, the axial deformation in the i^{th} fiber is given by:

$$\varepsilon_i = \varepsilon_0 + \phi y_i + \varepsilon_{ri} \qquad \text{Eq. 2}$$

where y_i is the distance between the plastic centroids of the analyzed fiber and the cross section; ε_0 is the axial deformation at the PC of the section; ε_{ri} is the deformation due to residual stresses (when considered); the respective curvature. In matrix notation, the variables ε_0 and ϕ are components of the deformation vector X. Numerically, it can be said that the equilibrium of the section is obtained when the following equation, written in matrix form, is satisfied:

$$\mathbf{F}(\mathbf{X}) = \mathbf{f}_{ext} - \mathbf{f}_{int} = \begin{bmatrix} N_{ext} \\ M_{ext} \end{bmatrix} - \begin{bmatrix} N_{int} \\ M_{int} \end{bmatrix} \cong \mathbf{0} \qquad \text{Eq. 3}$$

where the vector of external forces f_{ext} is given by the axial force N_{ext} and bending moment M_{ext}; and the terms N_{int} and M_{int} are the components of the internal force vector, f_{int}. The internal forces are obtained from the deformed configuration of the cross-section through classical integrals, given by:

$$N_{int} = \iint_{A_c} \sigma_c dA + \iint_{A_b} \sigma_b dA = \sum_{i=1}^{n_{fib,c}} \sigma_{ci} A_{ci} + \sum_{i=1}^{n_{fib,b}} \sigma_{bi} A_{bi}$$

$$M_{int} = \iint_{A_c} \sigma_c y dA + \iint_{A_b} \sigma_b y dA = \sum_{i=1}^{n_{fib,c}} \sigma_{ci} A_{ci} y_{ci} + \sum_{i=1}^{n_{fib,b}} \sigma_{bi} A_{bi} y_{bi}$$

Eq. 4

where $n_{fib,c}$ and $n_{fib,b}$ are the number of fibers in the concrete section and the number of fibers in the longitudinal bars, respectively, whereby A_i is the area of the fiber, and y_i is the position of the fiber in relation to the Plastic Neutral Line.

Eq. 3 is nonlinear and its solution is obtained here through the iterative Newton-Raphson process. Although it is efficient to start the iterative process with X=0, convergence is only achieved in the first iteration if external forces are null. Thus, for the next iteration, k+1, the strain vector is calculated as:[9]

$$\mathbf{X}^{k+1} = \mathbf{X}^k + \mathbf{F}'\left(\mathbf{X}^k\right)^{-1} \mathbf{F}\left(\mathbf{X}^k\right) \qquad \text{Eq. 5}$$

where F' is the Jacobian matrix of the nonlinear problem, that is:

$$\mathbf{F}' = \left(\frac{\partial \mathbf{F}}{\partial \mathbf{x}}\right) = \begin{bmatrix} f_{11} = \dfrac{\partial N_{int}}{\partial \varepsilon_0} & f_{12} = \dfrac{\partial N_{int}}{\partial \phi} \\ f_{21} = \dfrac{\partial M_{int}}{\partial \varepsilon_0} & f_{22} = \dfrac{\partial M_{int}}{\partial \phi} \end{bmatrix}$$

Eq. 6

After convergence of the iterative process, the degradation of axial and flexural stiffness is evaluated in relation to the updated position of the PC. The generalized stiffnesses, considering this degradation, are calculated from the terms of the Jacobine F' matrix, that is:

$$EI_T = f_{22} - \frac{f_{12} f_{21}}{f_{11}} \qquad \text{Eq. 7}$$

$$EA_T = f_{11} - \frac{f_{12} f_{21}}{f_{22}} \qquad \text{Eq. 8}$$

Full yield curve

The behavior of the reinforced concrete cross section, as presented in the previous

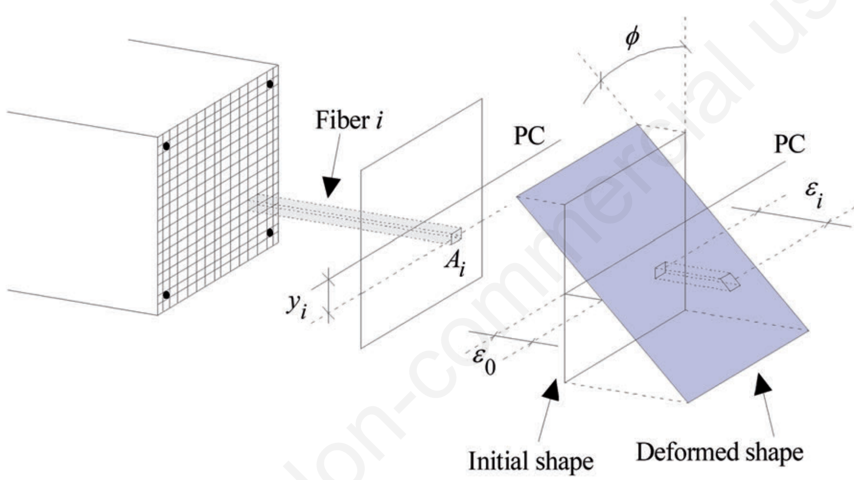

Figure 2. Linear strain field around x axis. PC, plastic centroid.

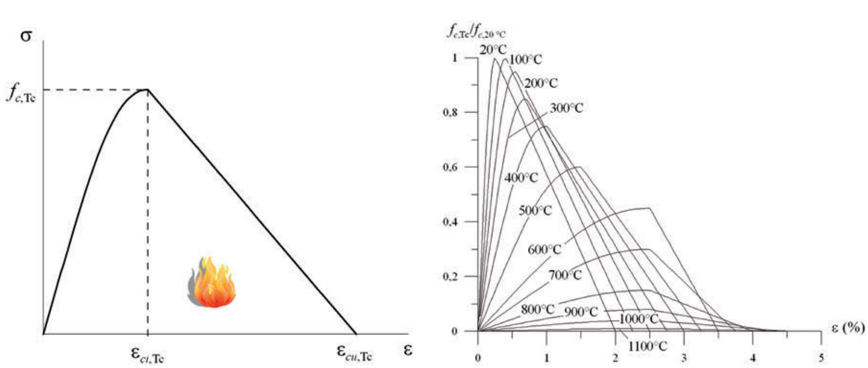

Figure 3. Concrete constitutive relationship under fire.

section, can be represented by the moment-curvature relationship. The maximum point in this relationship characterizes the section's resistant moment, so that the pair of values for the normal stress-bending moment (*N-M*) is taken as one of the points of the *N-M* interaction curve, which represents the resistance limits of the cross-section.[9] In order to accelerate the numerical simulations, these full yield curves are obtained for each cross-section of the members that make up the structural system, even before the structural analysis is performed. In the case of thermo-structural analysis, these curves must be constructed at each step in time. The flowchart of solution of the thermo-structural problem is illustrated in Figure 4.

Results and Discussion

Beams subject to uniform temperature

Presented herein are reinforced concrete beams that have been tested under fire.[21,22] The beams are illustrated in Figure 5 where the equilibrium paths at room temperature are shown for each of them. Experimental results of these beam tests are used by many authors mainly for the purpose of validating finite element formulations for structural analysis in fire situations. It is noteworthy that, we sought experimental tests where their results were reported in as much detail as possible to enable better modeling and numerical simulations in the present study.

Wu *et al.*[21] tests

This section provides the thermo-structural analysis of two of the three beam models tested by Wu *et al.*[21] The dimensions and loading details of these beams are illustrated in Figure 6. During the fire test, a slab was placed on the beams. The slab was 80 mm thick for Beam I and 120 mm thick for Beam III. In addition, a distributed load was applied to the top of the overlapping slab (Figure 6). The yield strength of the steel bars was taken equal to 240 MPa and the modulus of elasticity equal to 210 GPa. The compressive strength of concrete for Beams I and III was considered equal to 23.1 MPa and 27.1 MPa, respectively. The beams were subjected to the standard fire defined by the ISO 834-1[23] curve on the lower and lateral faces and the cross section was discretized into 342 linear quadrilateral finite elements.

Figure 7 provides the predicted temperature increase for the depths of 10 mm, 25 mm and 100 mm in the cross section. The results of the present study have been compared to the experimental results of Wu *et al.*[21] and the numerical results of Gao *et al.*,[24] who used a 3D finite element formulation using ABAQUS software. As can be seen from Figure 7, the results obtained here are close to those found in literature. The temperature at 100 mm from the underside is slightly lower in the first 40 minutes of fire exposure compared to the experimental result.

This difference may be related to the

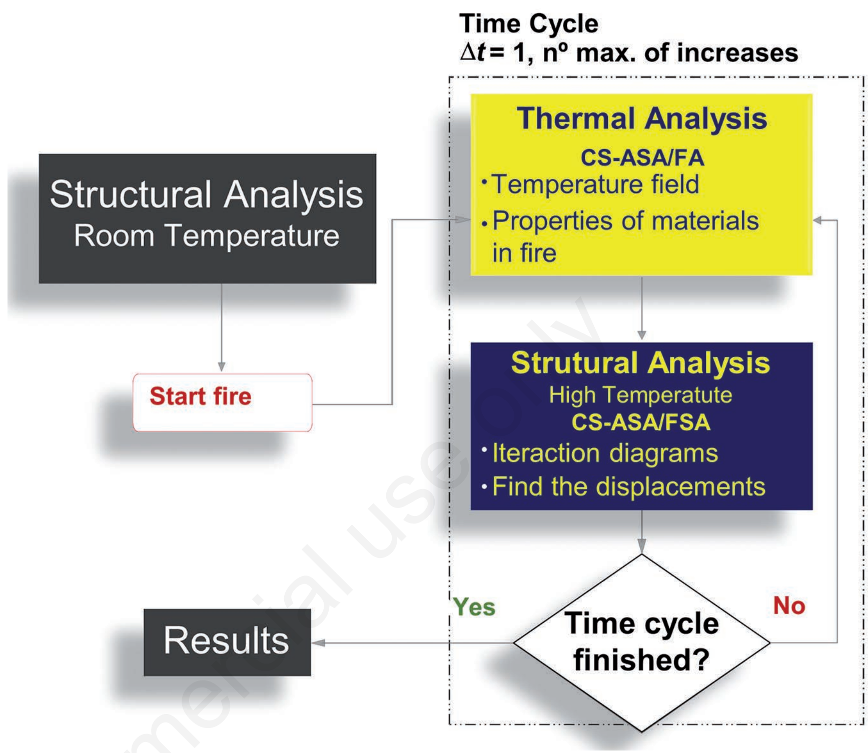

Figure 4. Thermal-structural problem solution. CS-ASA/FA, Computational System for Advanced Structural Analysis/Fire Analysis; CS-ASA/FSA; Computational System for Advanced Structural Analysis/Fire Structural Analysis.

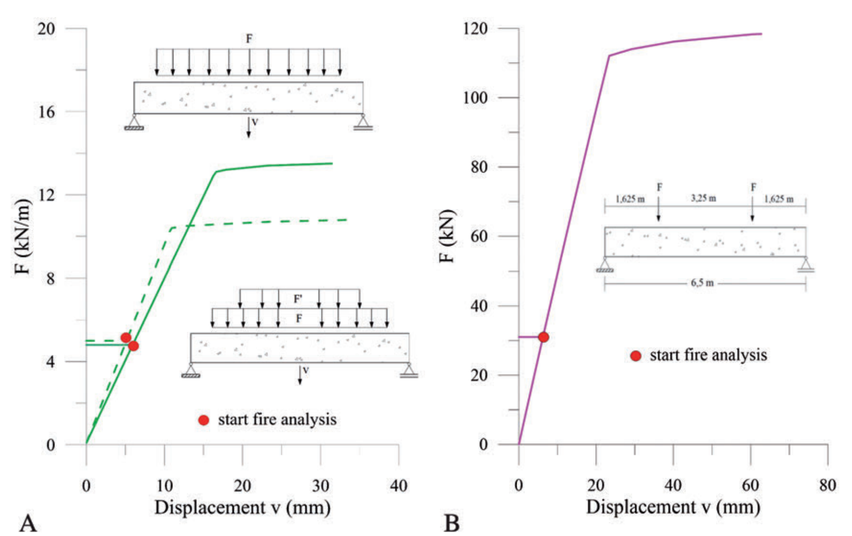

Figure 5. Beams and equilibrium path: A) Wu *et al.*[21] B) Dotreppe and Franssen.[22]

migration of moisture to the interior of the beam, as also observed by Gao *et al.*[24] Temperature underestimation has little effect on beam performance, which in this case is because the mechanical properties of concrete and steel remain almost unchanged as the temperature is still relatively low (about 100°C).

In both tests, the free span of the beams was 5100 mm and the maximum allowed deflection was therefore 255 mm (L/20), highlighted in the graphs of Figure 8. It is worth emphasizing that in the tests by Gao *et al.*,[24] the failure modes for Beams I and III were excessive local temperature increase and excessive deflection of the slab, respectively. In the graphs, the good agreement between the results of this

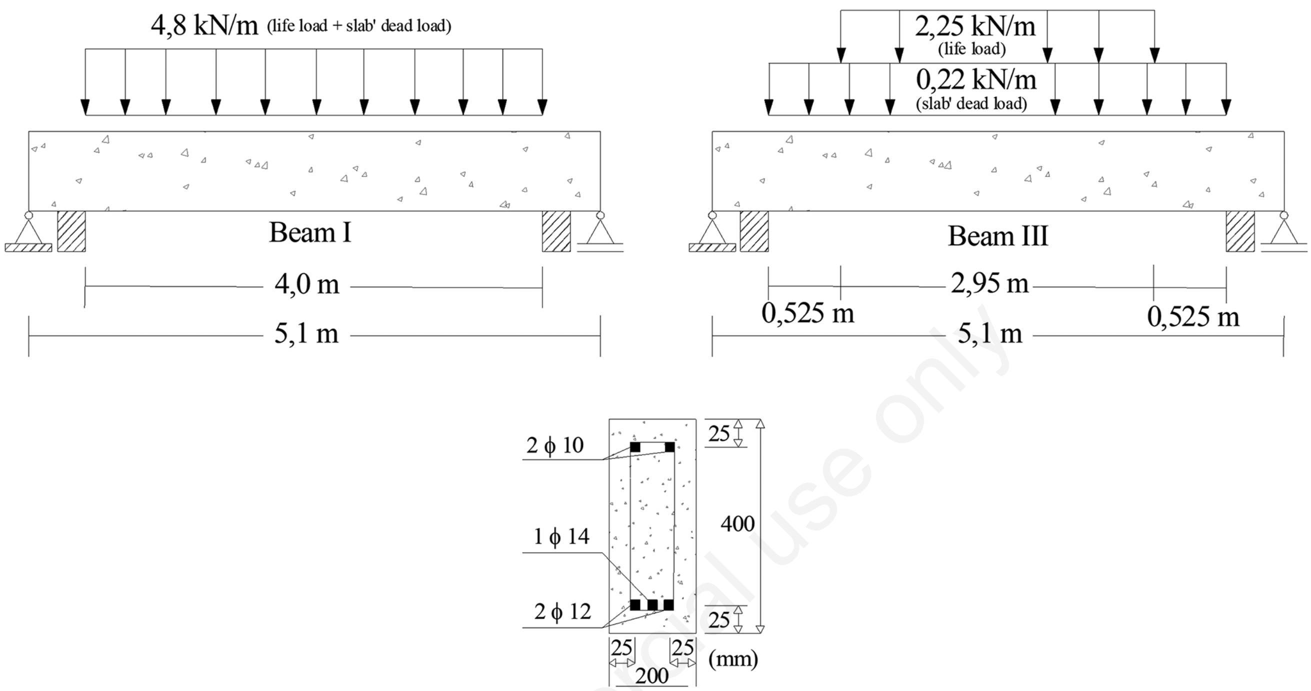

Figure 6. Details of Beams I and III in reinforced concrete.

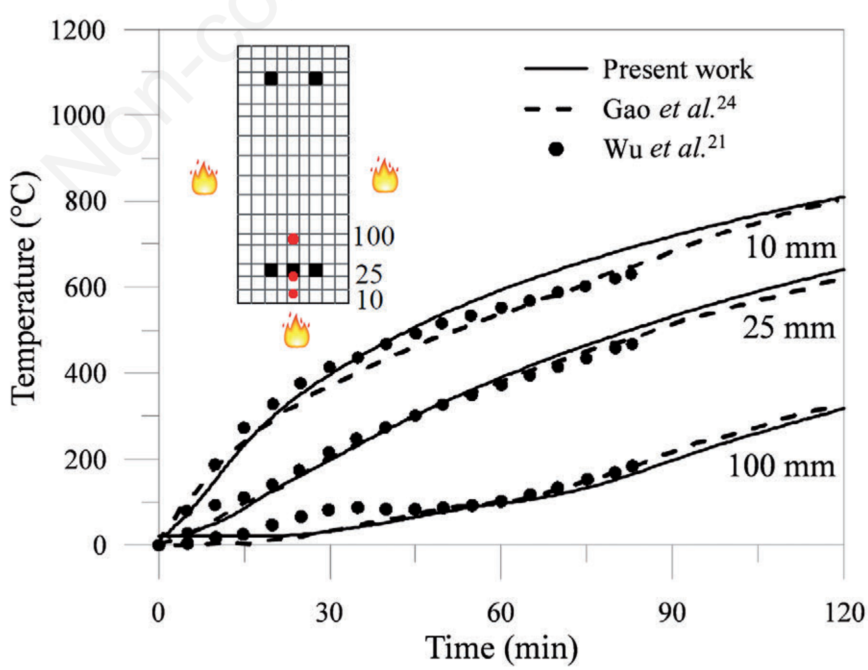

Figure 7. Temperature and time curves.

research and those found in literature can be observed.

Dotreppe and Franssen[22] tests

The other test modeled in this study was performed by Dotreppe and Franssen.[22] These researchers studied a simply supported reinforced concrete beam subjected to the standard fire curve[23] on three faces (Figure 9). The beam was initially subjected to a symmetrical loading consisting of two concentrated loads (F) of 32.5 kN each. The steel yield stress and modulus of elasticity were considered equal to 500 MPa and 210 GPa, respectively. The average strength of the concrete was taken equal to 25 MPa. For numerical analysis, the cross-section was discretized into 320 linear quadrilateral finite elements and the beam was divided into 12 finite elements.

The results obtained for the central steel bar temperature rise and the vertical displacement in the middle of the beam span are shown in Figure 10A and B, respectively. Again, in addition to the experimental results by Dotreppe and Franssen,[22] the numerical results by Gao *et al.*[24] are used for comparison. As can be seen from the graphs in Figure 10, the results obtained here are satisfactory over the entire duration of fire exposure.

Conclusions

The present study aimed to present and describe aspects of interest related to the development and subsequent application of a computer system capable of simulating the behavior of reinforced concrete beams in fire situation. The computer system implemented had already been successfully used in Barros *et al.*[14] for the analysis of steel structures under high temperature.

Through the analysis of the results presented and other studies also cited in the text, it is concluded that the computational modules developed and implemented can be used, in a satisfactory way, for the behavior analysis of reinforced concrete beams in fire situation. Based on the concept of concentrated plasticity, the numerical strategy adopted here considers the coupling of SCM and RPHM, which was able to accurately capture the true inelastic behavior of beams subjected to high temperatures. In addition, it provided the failure time and/or temperature close to those found in literature. This research also extends to the analysis of other reinforced concrete structural systems, and furthermore, to studies that are being directed toward the analysis of composite steel-concrete structures in fire condition.

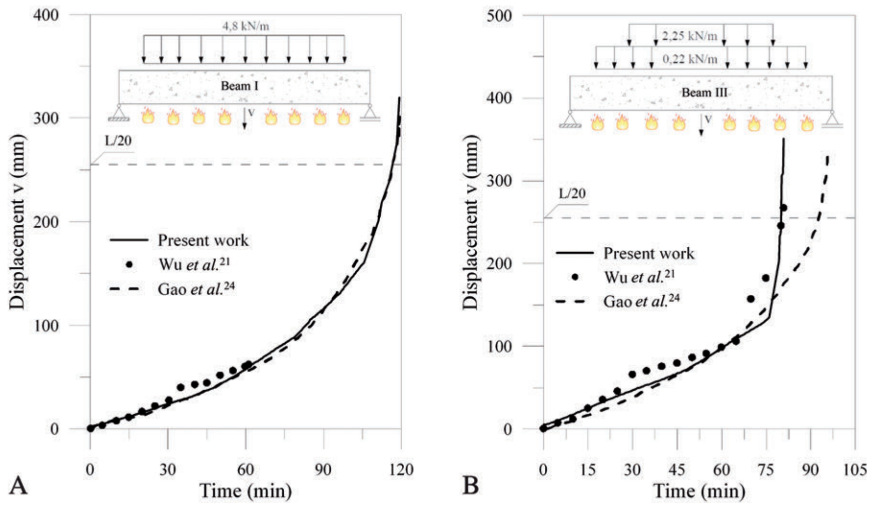

Figure 8. Vertical displacement × time. A) Beam I. B) Beam III.

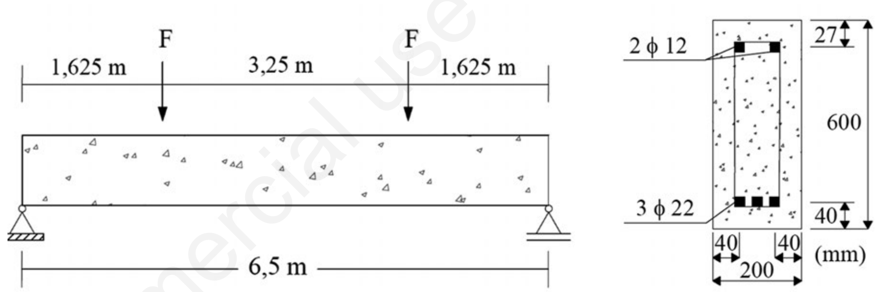

Figure 9. Details of the simply supported beam on reinforced concrete.

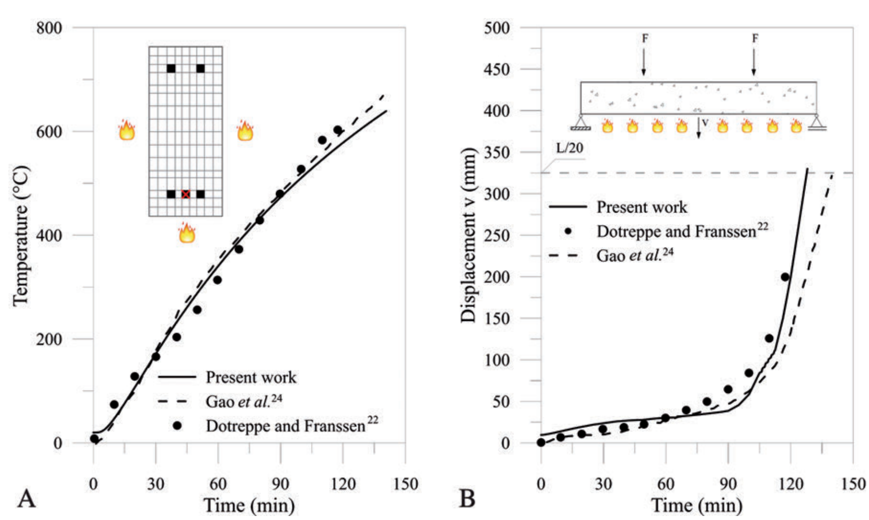

Figure 10. Thermo-structural analysis. A) Temperature in the central steel rebar. B) Vertical displacement × time curve.

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